Trinity College

WA Exams Practice Paper B, 2016

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	11	11	100	98	65
			Total	149	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

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Section One: Calculator-free **35% (51 Marks)**

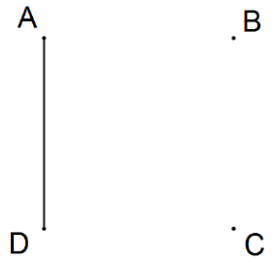
This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(6 marks)**

The adjacency matrix, M, for a graph with vertices A, B, C and D is $\begin{matrix} & A & B & C & D \\ A & 0 & 0 & 1 & 1 \\ B & 0 & 1 & 1 & 0 \\ C & 1 & 1 & 0 & 2 \\ D & 1 & 0 & 2 & 0 \end{matrix}$.

(a) Complete the graph for M in the space below. (3 marks)



(b) The matrices M^2 and M^3 are $M^2 = \begin{bmatrix} 2 & 1 & 2 & 2 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 6 & 1 \\ 2 & 2 & 1 & 5 \end{bmatrix}$ and $M^3 = \begin{bmatrix} 4 & 3 & 7 & 6 \\ 3 & 3 & 7 & 3 \\ 7 & 7 & 5 & 14 \\ 6 & 3 & 14 & 4 \end{bmatrix}$.

(i) State the number of walks from C to A of length one. (1 mark)

(ii) State the number of walks from C to B of length two. (1 mark)

(iii) State the number of walks from D to C of length three. (1 mark)

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Question 2 **(5 marks)**

A sequence of numbers is described by the recursive equation $T_{n+1} = T_n - 8, \quad T_4 = 35$.

(a) Determine T_6. (1 mark)

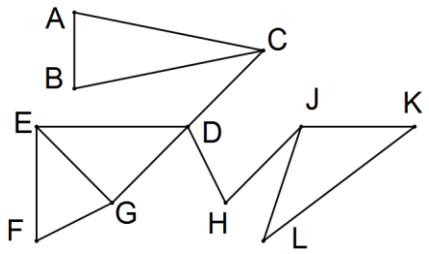
(b) Determine T_1. (1 mark)

(c) State a rule for the n^{th} term of this sequence. (2 marks)

(d) Determine T_{1001}. (1 mark)

Question 3 **(8 marks)**

(a) A simple connected graph is shown below.



(i) List all the edges in the connected graph below that are bridges. (2 marks)

(ii) State the number of vertices in the graph that have an odd degree. (1 mark)

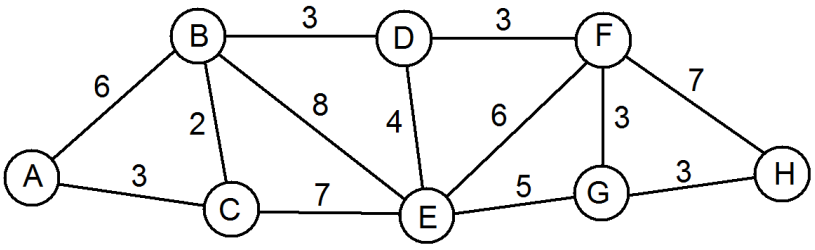
(iii) The graph will have a semi-Eulerian trail if one more edge is added to it. Briefly explain what a semi-Eulerian trail is and state a suitable pair of vertices that must be joined with an edge for this to occur.

(3 marks)

(b) Show use of Euler's formula to determine the number of edges in a graph with four vertices and one face and sketch a planar graph with these properties that also has no vertices with an even degree. (2 marks)

Question 4 **(8 marks)**

The weighted graph below shows the cost (in hundreds of dollars) to transport high-security items directly from one depot of a freight company to adjacent depots.



(a) How many depots are shown on the graph? (1 mark)

(b) Between which two adjacent depots is the transport cost the highest and what is the cost? (2 marks)

(c) Determine the minimum cost of transporting a high-security item through from A to H, listing the route the item must take. (3 marks)

(d) A driver leaves E, travels directly to F and then visits all the other depots before returning to E. Explain whether or not this walk could be a Hamiltonian cycle. (2 marks)

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Question 5 **(8 marks)**

The table below shows, to the nearest cent, the value of \$1 000 invested for one year at different interest rates and compounding periods.

	Annual interest rate					
Interest Compounded	*1%*	*2%*	*3%*	*4%*	*5%*	*6%*
Annually	1 010.00	1 020.00	1 030.00	1 040.00	1 050.00	1 060.00
Half-Yearly	1 010.03	1 020.10	1 030.23	1 040.40	1 050.63	1 060.90
Quarterly	1 010.04	1 020.15	1 030.34	1 040.60	1 050.95	1 061.36
Monthly	1 010.05	1 020.18	1 030.42	1 040.74	1 051.16	1 061.68
Daily	1 010.05	1 020.20	1 030.45	1 040.81	1 051.27	1 061.83

(a) State the value of \$1 000 invested at 3% pa compounded monthly after one year. (1 mark)

(b) How much interest will be earned when \$1 000 is invested at 5% pa compounded every six months for one year? (1 mark)

(c) \$1 000 was invested at 4% pa for one year. If \$40.60 interest was earned, what was the compounding period? (1 mark)

(d) How much extra interest is earned by compounding the interest monthly instead of half-yearly when \$1 000 is invested at 6% pa for one year? (1 mark)

(e) State the value, to the nearest dollar, of \$10 000 invested at 6% pa compounded monthly after one year. (2 marks)

(f) The twelfth term of the recursive rule $T_{n+1} = r \times T_n$, $T_0 = a$, would also be the answer to (e). State the values of a and r in this rule. (2 marks)

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Question 6 **(8 marks)**

A company has a fleet of four mobile coffee trucks that it plans to send to four venues the following week. The table below shows the average weekly profit (in hundreds of dollars) made by each truck at these venues in the past.

	Venue 1	Venue 2	Venue 3	Venue 4
Truck A	50	65	55	65
Truck B	45	55	40	55
Truck C	35	50	65	70
Truck D	40	50	45	55

(a) Briefly explain why the Hungarian algorithm cannot be used with the numbers as shown in the table to determine the allocation of trucks to maximise the total profit. (1 mark)

(b) Create a matrix that can be used with the Hungarian algorithm by subtracting all the numbers in the table from 70. (2 marks)

(c) What do the numbers in the matrix in (b) show? (1 mark)

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(d) Show use of the Hungarian algorithm to determine the allocation of trucks to the different venues to maximise the profit, and state what this maximum profit is. (4 marks)

See next page

Question 7 **(8 marks)**

The completion times and immediate predecessors for all the tasks (A, B, C, D, E, F, G, H and J) that are involved in a project are listed here:

- A (12 days), B (20 days) and C (19 days) have no predecessors
- D (11 days) and G (24 days) can begin once A has been completed
- F (8 days) can begin once B and D have been completed
- E (9 days) and H (18 days) can begin once C has been completed
- J (10 days) can begin once E and F have been completed

(a) Display this information as a project network. (4 marks)

(b) State the critical path and minimum completion time for this project. (2 mark)

(c) Tasks G and H are delayed by x and y days respectively, where x and y are positive integers, but the minimum completion time and the **unique** critical path are unchanged.

Determine the maximum value of $x + y$. (2 marks)

End of questions

Additional working space

Question number: _____